

SPECIALIST MATHEMATICS Teach Yourself Series

Topic 11: Vector Calculus

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Vector Calculus

The objective of studying this topic is to differentiate and antidifferentiate a vector function to analyse the motion of a particle along a curve. It includes understanding the concept of position vectors as a function of time and to be able to represent the path of a particle in two and three dimensions (as a vector function). This branch of mathematics helps us to understand motion concepts in Physics.

Vector equations and Position vectors as a function of time As it appears in Unit 4

- These are used to describe the motion of particles.
- The paths described are the same but the particles are at different locations at a given time.
- Vector equations can be converted to Cartesian equations by substituting the i component as x and the j as y, and eliminating r and θ to get a relation between x and y only.
- Position vectors tell us about the path of an object and its motion in time.
- The position vector of a particle at time *t* in two dimensions is given by

$$r(t) = x(t) i + y(t) j$$

• The parametric form of the path of a particle is given by

$$x = x(t), y = y(t)$$

Eg. A particle moves so that its position vector r at time t, $t \ge 0$ is

 $r = (2\cos(3t) - 1)i + \sin(3t)j$. Determine the Cartesian equation of the path.

$$\cos^{2}(3t) + \sin^{2}(3t) = 1$$
$$\frac{(x+1)^{2}}{4} + \frac{y^{2}}{1} = 1$$

An ellipse with centre ($^{-}1$, 0), a = 2, b = 1

Review Questions

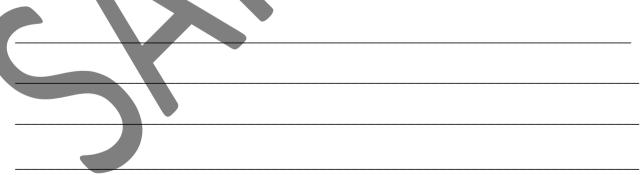
1. Find the Cartesian equation for the path of an object with the position vector

a. $r(t) = \ln (2t) i + (4t - 1) j$.

b. $r(t) = (3e^{-2t} + 1)i + (2e^{-3t} - 1)j$



c. $r = (2\cos(3t) - 1)i + \sin(3t)j$



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2.	find the distance of the body described by the following position vectors from the origin at the time	S
	sted, using distance = $\sqrt{r \cdot r} = r $.	

~ ` / ~ ~ • • • / •	a.	$r(t) = 2t^2 i$	$+\log(10t) j$	at $t = 1$
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b.	$r(t) = 4e^{-2t} j - 2t^{-1} j$ at $t = \frac{1}{2}$			





Solutions to Review Questions

1.

a.

$$x = \ln(2t), \quad e^x = 2t, \quad t = \frac{e^x}{2}$$
$$y(x) = 4t - 1$$
$$= 4\left(\frac{e^x}{2}\right) - 1$$
$$= 2e^x - 1$$

b. Let
$$x = 3e^{-2t} + 1$$

$$x-1 = 3e^{-2t}$$

$$e^{-2t} = \frac{x-1}{3}$$

$$\left(e^{-2t}\right)^{\frac{3}{2}} = \left(\frac{x-1}{3}\right)^{\frac{3}{2}}$$

$$e^{-3t} = \left(\frac{x-1}{3}\right)^{\frac{3}{2}}$$

$$y = 2e^{-3t} - 1$$

$$= 2\left(\frac{x-1}{3}\right)^{\frac{3}{2}} - 1$$

c.
$$r = (2\cos(3t) - 1) j + \sin(3t) j$$

 $x = 2\cos(3t) - 1$, $y = \sin(3t)$
 $x + 1 = 2\cos(3t)$
 $\cos(3t) = \frac{x+1}{2} \sin(3t) = \frac{y}{1}$
 $\cos^2(3t) + \sin^2(3t) = 1$
 $\frac{(x+1)^2}{4} + \frac{y^2}{1} = 1$

2.

a.
$$|\underline{r}| = \sqrt{\underline{r} \cdot \underline{r}}$$
$$= \sqrt{2^2 + (\log(10))^2}$$
$$= \sqrt{4+1}$$
$$= \sqrt{5}$$

b.
$$|r| = \sqrt{(4e^{-1})^2 + \left(-2 \times \left(\frac{1}{2}\right)^{-1}\right)^2}$$

$$= \sqrt{\frac{16}{e^2} + (-4)^2}$$

$$= \frac{1}{e}\sqrt{16 + 16e^2}$$

3.

a.
$$y(t) = \frac{d r(t)}{dt} = \frac{d}{dt} [(3t^2 - \sin(\pi t)) i + (e^{-t} + 2t) j]$$

= $(6t - \pi \cos(\pi t)) i + (e^{-t} + 2t) j$

b.
$$y(2) = (6 \times 2 - \pi \cos(2\pi)) \dot{i} + (-e^{-2} + 2) \dot{j}$$

= $(12 - \pi) \dot{i} + (2 - e^{-2}) \dot{j}$

c. Average velocity =
$$\frac{r(2) - r(0)}{2}$$

$$= \frac{(3\times2^{2})i + (e^{-2} - 1 + 2\times2)j}{2}$$

$$= 6i + \frac{e^{-2} + 3}{2}j$$

4.

a.
$$r(0) = (3e^{-2(0)} + 1)i + (2e^{-3(0)} - 1)j = 4i + j$$

$$v(t) = (-6e^{-2t})i + (-6e^{-3t})j$$

$$v(0) = (-6e^{-2(0)})i + (-6e^{-3(0)})j = -6i - 6j$$

$$a(t) = (12e^{-2t})i + (18e^{-3t})j$$

$$a(0) = 12i + 18j$$